



IFFERENTIAL EQUATION

ifferential equation is an equation that involves independent and dependent values and the derivatives of the dependent variables.

💵 TYPES OF DIFFERENTIAL EQUATION 📗

ORDINARY DIFFERENTIAL EQUATION

If the differential coefficients have reference to a single independent variable only.

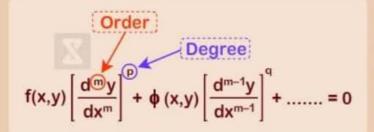
Eg:
$$\frac{d^2y}{dx^2} - \frac{2dy}{dx} + \cos x = 0$$

PARTIAL DIFFERENTIAL EQUATION

If there are two or more independent variables.

Eg:
$$\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$$

ORDER AND DEGREE OF DIFFERENTIAL EQUATION 🕪



Order: is the highest derivative.

Degree: is the exponent of the highest derivative.

SOLVING DIFFERENTIAL EQUATION

ELEMENTARY TYPE OF 1ST ORDER AND 1ST DEGREE

SEPERATION OF VARIABLES METHOD:

- Step 1 Move all the y terms (including dy) to one side and all the x terms (inculding dx) to the other side.
- Step 2 Intergrate one side with respect to 'y' and the other side with respect to 'x'.
- Step 3 Simplify it.

SEPERATION OF VARIABLES TYPES:

Type-1: Equation of the form
$$f(x)dx = -g(y)dy \Longrightarrow \int f(x)dx + \int g(y)dy = C$$

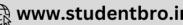
Type-2: Equation of the form
$$\frac{dy}{dx} = f(ax + by + c)$$
; $b \neq 0 \Longrightarrow put ax + by + c = t \& reduce to Type 1$

Type-3: Equation of the form
$$\frac{dy}{dx} = \frac{a_1x + b_1y + c_1}{a_2x + b_2y + c_2}$$
 { if $a_2 + b_1 = 0$ } Cross multiply and note the perfect differential d(xy).

(a)
$$x = r \cos \theta$$
; $y = r \sin \theta$; $x^2 + y^2 = r^2$; $\frac{y}{x} = \tan \theta$; $x dx + y dy = r dr$; $x dy - y dx = r^2 d\theta$

(b)
$$x = r \sec \theta$$
; $y = r \tan \theta$; $x^2 - y^2 = r^2$; $\frac{y}{x} = \sin \theta$; $xdx - ydy = rdr$; $xdy - ydx = r^2 \sec \theta d\theta$





HOMOGENEOUS DIFFERENTIAL EQUATION



T-(1) An equation of the form $\frac{dy}{dx} = \frac{f(x, y)}{g(x, y)}$ (where f(x, y) and g(x, y) are homogeneous functions of x

and y and of same degree) To solve put y = tx or x = ty

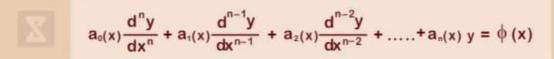
$$y = tx \text{ or } x = ty$$



 $\frac{a_1}{a_2} \neq \frac{b_1}{b_2}$. Substitution: x = u + h and y = v + k then reduce differential equation to Homogeneous **Differential Equation T-1**

LINEAR DIFFERENTIAL EQUATION

A differential equation is said to be linear if the dependent variable and its differential coefficients occur in the first degree only and are not multiplied together.



where $a_0(x)$, $a_1(x)$, $a_2(x)$, $a_n(x)$ are called coefficient of Differential Equation.

Linear differential equation of first order

$$\frac{dy}{dx} + P(x)y = Q(x)$$

To solve

Calculate: Integrating factor (I.F.) = $e^{\int P(x)dx}$

Solution: $y(I.F.) = \int Q(x)(I.F.)dx + C$

Bernoulli's Equation

Equation reducible to linear differential equation

Equation of the form
$$\left[\frac{dy}{dx} + P(x)y = Q(x)y^n\right]$$

To solve divide by ynand substitute y1-n = t

$$\frac{dt}{dx}$$
 + (1 - n) P(x)t = Q(x)(1 - n)

Now solve as 1st order Linear Differential Equation

📲 SOME IMPORTANT EXACT DIFFERENTIALS 🕪

$$\frac{xdy - ydx}{x^2} = d\left(\frac{y}{x}\right)$$

$$\frac{xdy - ydx}{y^2} = d\left(-\frac{x}{y}\right)$$

$$\frac{xdy + ydx}{xy} = \frac{d(xy)}{xy} = d(\ln xy)$$

$$\frac{xdy - ydx}{xy} = d\left(\ln\left(\frac{y}{x}\right)\right)$$

$$\frac{ydx - xdy}{xy} = d\left(\ln\frac{x}{y}\right)$$

$$\frac{xdy + ydx}{x^2y^2} = d\left(-\frac{1}{xy}\right)$$

•
$$\frac{2(xdx + ydy)}{x^2 + y^2} = [\ln(x^2 + y^2)]$$
 • $\frac{dx + dy}{x + y} = d(\ln(x + y))$

$$\frac{xdy - ydx}{x^2 + y^2} = d\left(\tan^{-1}\frac{y}{x}\right)$$

$$d\left(\frac{e^{y}}{x}\right) = \left(\frac{xe^{y}dy - e^{y}dx}{x^{2}}\right)$$

$$d\left(\frac{e^x}{y}\right) = \left(\frac{ye^xdy - e^xdx}{y^2}\right)$$

$$\frac{dx + dy}{x + y} = d \left(\ln(x + y) \right)$$

